Use of Chilled Mirror /Dew-Point Method (WP4C) to Develop Soil Water Characteristic Curve for Coastal Silty Sand with a Comparison of the Existing Analytical Models

A.A.S. Kaushalya, M.A. Pallewattha, F.S.A. Ansar and U.P. Nawagamuwa

Abstract: Soil Water Characteristic Curve (SWCC) is one of the methods to study the behaviour of unsaturated soil. This study presents the SWCC for Sri Lankan coastal silty sand developed using chilled mirror dew-point technique WP4C and compares the performance of a few prediction models. SWCC was also developed experimentally using pressure plate and suction-moisture sensors, to verify the results obtained using WP4C. The fitted SWCC curve was developed using Van Genuchten’s (VG) model. The prediction models of Arya and Paris (1981), Aubertin et al. (2003), and Wang et al. (2017) were used to compare the performance using their own suggested values and the more suitable values for this study as the model parameters which were estimated using the least square method. This study shows that chilled mirror dew-point technique which is incorporated in WP4C instrument can be used as a fast and reliable experimental method to develop the SWCC for sandy soils compared to other experimental methods. The Wang et al. (2017) model is better suited for estimating the SWCC through analytical methods for this soil type, using either the recommended values in this study or proposed values by the authors for the modal parameters.

Keywords: Unsaturated coastal silty sand, Soil Water Characteristic Curve (SWCC), Dew point chilled mirror technique, WP4C

1. Introduction

Mechanical and hydraulic properties of unsaturated soils are predicted using the Soil Water Characteristic Curve (SWCC), initially developed in agriculture science to represent the water storage capacity of a specific material [1]. SWCC represents the relationship between matric suction and the degree of saturation that can be replaced by gravimetric or volumetric water content. SWCC depends on the soil mineralogy, grain and pore-size distribution, porosity, surface tension, texture, fabric, particle shape, wetting-drying cycles, contact angle, and entrapped air [2].

SWCC has three phases, namely, 1) capillary saturation zone (boundary effect zone), 2) desaturation zone (transition zone), and 3) the residual saturation zone (residual zone), as shown in Figure 1. The soil remains saturated even when the pore water is in tension due to capillary forces in the capillary saturation zone. This zone ends at the air entry value of the soil. In the desaturation zone, the water is displaced by air within the pores, and the zone ends with residual water content. In the residual saturation zone, the water is strongly adsorbed onto the soil particles and flows as vapour [1].

SWCC can be developed by conducting experimental procedures with pressure plates, tensiometers, filter paper, moisture-suction sensors, etc. Mahannopkul & Jotisankasa [3] experimentally developed the SWCC for clayey sand in Southern Thailand using three types of equipment varying according to the matric suction value. Isosteric Humidity Control has been used for matric suction values ranging from 0 to 0.5 MPa, and Isopestic Humidity Control has been used for matric suction values ranging from 0.5 to 2.0 MPa. At each condition, the water content was measured using filter paper at 240, 480, 720, 960 and 1200 hours. The SWCC was then fitted with the Van Genuchten (VG) model, which is given by the following equation:

\[ \theta = \theta_r + (\theta_s - \theta_r) \left( \frac{s}{s_c} \right)^{m} \left[ 1 - \left( \frac{s}{s_c} \right)^{m} \right]^{\alpha} \]

where \( \theta \) is the volumetric water content, \( \theta_r \) is the residual water content, \( \theta_s \) is the saturated water content, \( s \) is the matric suction, \( s_c \) is the air entry value, \( m \) is the shape parameter, and \( \alpha \) is the exponent parameter.

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from $10^3$ to $10^5$ kPa, whereas Tensiometer ranges from $10^{-1}$ to $10^{-2}$ kPa. The Pressure Plate has covered the range in between. Croney & Coleman [5] observed the behaviour of incompressible and compressible soils by developing SWCCs using the pressure plate. Developing SWCC using the apparatus as mentioned earlier is a time and labour-consuming task. The chilled mirror or dew-point method was developed to measure the total suction in the soil to a high suction range using the water activity meter [6]. Moreover, this device was further developed as WP4C- Dew Point Potentiometer Meter [ASTMD 6836-02(2008)e2], which measures water suction quickly, precisely, and consistently. Hence, WP4C can be used to develop the SWCC. Although WP4C is a convenient equipment, the development of SWCC utilizing this instrument is rare in the literature [4]. Thus, verifying the results obtained from WP4C using other apparatus will increase the reliability of the output.

Furthermore, prediction models have been developed to estimate the SWCC by using basic soil properties such as grain size distribution, void ratio, and density. Arya & Paris [7], Haverkamp & Parlange [8], Arya & Dierolf [9], Fredlund [10], Aubertin et al. [11], and Wang et al. [12] developed different models to predict the SWCC and these models are based on different theories and fundamentals. As a result, they have some restrictions and may yield to inconsistent results.

This paper presents the SWCC of Sri Lankan coastal silty sand developed using Dew-point chilled mirror technique, WP4C, and compares the performance of a few prediction models. Further, the SWCC has been developed experimentally using two different apparatuses, pressure plate and suction-moisture sensors, to verify the results obtained using the WP4C. Arya and Paris [7], Aubertin et al. [11], and Wang et al. [12] have been used to compare the performance of prediction models.

2. Literature Review

2.1 Developing SWCC using Experimental Methods

2.1.1 Pressure Plate Apparatus

The pressure plate apparatus developed by Soil Moisture Equipment Cooperation can be used to develop the SWCC of soil by following the pressure plate test as per ASTM D2325 standard. The main components of the pressure plate apparatus are the high-air-entry ceramic disk and pressure chamber. The ceramic plate should always be in contact with water in a compartment below the disk and maintained in a saturated condition. This apparatus can be used with a high matric suction range (i.e., up to 1500 kPa). Soil specimens are placed on the ceramic plate and pressurized with the desired matric suction such that it does not exceed the air entry value of the ceramic plate. The water starts draining through the disk due to the applied pressure, and the soil specimen reaches the desired matric suction at the equilibrium stage. The corresponding volumetric water content can be obtained at this stage by measuring the weight of the specimen. The SWCCs have been developed using pressure plate apparatus in many countries [4].

2.1.2 WP4C

WP4C can be used to determine the SWCC in an extensive range. It uses the dew point chilled mirror technique to measure the soil suction and consists of a sealed chamber with a fan, a photoelectric cell, a mirror, and an infrared thermometer, as shown in Figure 2. The soil specimen can be prepared in a plastic container or stainless steel, ensuring the soil does not spill and contaminate the sample chamber. The specimen is then placed in the chamber and thermodynamically equilibrated with the environmental properties of the chamber. The first sign of condensation on the mirror is indicated by a photoelectric cell.

![Figure 1 - Phases of Soil Water Characteristic Curve [4]](image-url)
2.1.4 Van Genuchten’s Model
SWCCs developed using experimental procedures can be fitted into an S-curve according to Van Genuchten’s (VG) model using Equation 3 [14].

$$\theta = \theta_s + \frac{\theta_w - \theta_s}{[1 + (\alpha h)^n]^m}$$ …(3)

where $\theta$ is the volumetric moisture content, $h$ is matric water suction (kPa), $\theta_s$ is the soil saturated moisture content, $\theta_w$ is the soil residual moisture content, $\alpha$ is a scale parameter inversely proportional to mean pore diameter (cm$^{-1}$), $m$ and $n$ are the shape parameters of soil water characteristics with $m=(n-1)/n$, $0 < m < 1$.

2.2 Developing SWCC using Prediction Models
2.2.1 Arya and Paris (1981) Model
Arya & Paris [7] developed the SWCC using the Particle Size Distribution (PSD) of a soil, particle density, and bulk density. The cumulative PSD is divided into “n” fractions, and each segment’s pore size is determined. The pore volume of each fraction ($V_{vi}$) is calculated using the solid mass per unit sample, particle density, and void ratio. The accumulated volumetric water content ($\theta_{vi}$) stored up to the fraction “i” is given by Equation 4 where $\rho_{dry}$ is the dry density.

$$\theta_{vi} = \sum_{i=1}^{n} \frac{V_i}{V} = \sum_{i=1}^{n} V_{vi} \rho_{dry}$$ …(4)

Matric suction can be estimated using Laplace’s equation, a relationship between surface tension and the mean pore radius ($r_i$), the latter related to the mean particle radius ($R_i$) by Equation 5. Here, $n_i$ is the number of particles.

$$r_i = R_i \sqrt{\frac{4 \pi n_i^{1-n}}{6}}$$ …(5)

An empirical parameter $\alpha$ has been introduced to account for the non-spherical nature of the particles. Change in $\alpha$ translates the entire SWCC along the matric suction axis. This study suggests a value in between 1.35 – 1.40 for $\alpha$, which can be obtained by an iterative method such that the value of $|\ln(u_a - u_w)_{\text{measured}} - \ln(u_a - u_w)_{\text{calculated}}|$ becomes minimum.

2.2.2 Aubertin et al. (2003) Model
Aubertin et al. [11] used the primary soil properties to predict the SWCC. Capillarity and particle adhesion have been considered to predict the SWCC using the basic geotechnical
properties of coarse and fine-grained materials. The following equation has been used to evaluate the volumetric water content for a given matric suction. The degree of saturation ($S_r$) is computed as per Equation 6.

$$S_r = \frac{\theta}{n} = S_a + S_a^* (1 - S_a)$$  \hspace{1cm} \ldots(6)$$

Here $\theta$ is the volumetric water content, and $S_a^*$ is the degree of saturation caused by the adhesion, and $S_a$ is the degree of saturation associated with capillary forces which depend on the coarse and clayey nature of the soil. The capillary rise of granular materials is calculated using $C_u$ (coefficient of uniformity), $e$ (void ratio), and $D_{10}$ (soil diameter at which 10% of the soil weight is finer).

2.2.3 Wang et al. (2017) Model

Van Genuchten's water retention model and basic soil properties were used to develop the SWCC by Wang et al. [12]. The degree of saturation is measured using Equation 7 in this method. Here, $u_a$ and $u_w$ are pore air pressure and pore water pressure, respectively. $a$ is a parameter related to air entry value, and $n$ is a parameter associated with the slope of the water retention curve. Parameters $n$ and $a$ are dependent on $C_u$ (coefficient of uniformity) and $D_{60}$ (soil diameter at which 60% of the soil weight is finer). Further, two different constants $C_1$ and $C_2$ depending on the soil material, have been introduced to evaluate the model parameters as mentioned above.

$$S = \left[ 1 + \left( \frac{u_a - u_w}{a} \right)^n \right]^{1/n-1}$$ \hspace{1cm} \ldots(7)$$

2.2.4 Available Literature on the Comparison of Prediction Models

Different research has focused on comparing the available prediction models with experimental data to evaluate the reliability of the prediction models. Alves et al. [2] focused on evaluating the performance of prediction models for the coefficient values proposed by the authors ([7], [9], [11] and [12]) using 19 glass beads varying in particle size distribution, density and porosity. According to the above study, the reliability of the models was evaluated using regression analysis, and it was found that Wang et al. [12] model predictions were more accurate as the model was originally developed for sands. Further, Arya & Paris [7], and Arya & Dierolf [9] model predictions overestimated the matric suction.

3. Methodology

3.1 Soil Study

Soil classification was conducted according to the Unified Soil Classification System (USCS), and the soil was identified as a well-graded silty SAND (SW-5M). Table 1 summarizes the results of basic soil tests conducted according to ASTM standards, and the PSD curve of the soil is shown in Figure 3. The sample dry density was considered to be 1500 kg/m$^3$ (i.e. 80% of maximum dry density) for experimental procedures.

![Figure 3 - Particle Size Distribution Curve](image)

### Table 1 - Basic Soil Properties

<table>
<thead>
<tr>
<th>Soil parameter</th>
<th>Results</th>
<th>Test conducted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percentage of gravel</td>
<td>0.3%</td>
<td>ASTM D422-63</td>
</tr>
<tr>
<td>Percentage of sand</td>
<td>87.8%</td>
<td></td>
</tr>
<tr>
<td>Percentage of fines</td>
<td>11.9%</td>
<td></td>
</tr>
<tr>
<td>Coefficient of uniformity ($C_u$)</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>Coefficient of curvature ($C_c$)</td>
<td>1.36</td>
<td></td>
</tr>
<tr>
<td>Liquid Limit</td>
<td>17%</td>
<td>ASTM D4318-83</td>
</tr>
<tr>
<td>Plasticity Index</td>
<td>Nearly zero</td>
<td>ASTM D4318-83</td>
</tr>
<tr>
<td>Specific gravity</td>
<td>2.67</td>
<td>ASTM D854-83</td>
</tr>
<tr>
<td>Organic content</td>
<td>1.28%</td>
<td>ASTM D2974 - 71</td>
</tr>
</tbody>
</table>

*Average in-situ dry density was measured at the borrow site

3.2 Development of SWCC using Experimental Methods

SWCC was experimentally developed using a pressure plate, suction-moisture sensors and WP4C in this study. All experiments were conducted in the drying path, and the sample dry density was maintained near 1500 kg/m$^3$...
which is the in-situ dry density of the soil in the area where the samples were collected.

3.2.1 Pressure Plate
Twelve samples were prepared in the retaining rings (refer to Figure 4) with an internal diameter of 47 mm and a height of 10 mm. The sample was compacted using a soil compactor and a collar (Figure 4). The air pressure \((u_a)\) was applied to the apparatus from 100 kPa to 300 kPa, and water pressure \((u_w)\) was maintained at 100 kPa. Therefore, the specimens were brought to equilibrium under matric suction values from 0 to 200 kPa. The detailed experimental procedure is described in Section 2.1.1.

Figure 4 - Sample Preparation for Pressure Plate Apparatus (a) Collar (b) Compactor (c) Ring (d) Pressure Plate Apparatus

3.2.2 WP4C
The samples were prepared in stainless steel sample cups to ensure quick equilibrium at sample chamber temperature and kept at half the height of the sampler to reduce the possibility of spilling sample material and contaminating the sample chamber. WP4C was allowed to warm up for 15 to 30 minutes to achieve reliable readings. The instrument was calibrated using the 0.50 mol/kg KCl salt standard before the commencement of the experiment. Furthermore, the difference between the sample and the block temperature \((T_s-T_b)\) was maintained between -0.5 and 0 °C. The detailed experimental procedure is described in Section 2.1.2. Moreover, the results were obtained in the range of 0 to 300 MPa for the total suction.

A relationship was developed between the Electrical Conductivity (EC) of soil extract and gravimetric moisture content as shown in Figure 5 to estimate the EC of the saturation extract of the soil. The best-fit line had a sum of squared residual (SSR) value of 0.941. The osmotic and matric suctions were calculated using Equation 1 and Equation 2.

![Figure 5 - Relationship between Electrical Conductivity and Gravimetric Moisture Content of the Soil](image)

3.2.3 Suction - Moisture Sensors
The soil samples were prepared in a wooden box with internal dimensions of 300 mm (W) x 300 mm (L) x 300 mm (H). The suction-moisture sensors were installed at a depth of 150 mm, and continuous data were obtained for four months using the sensors.

3.2.4 Curve fitting using Van Genuchten's Model
The data obtained from the above instruments were utilized to develop the SWCC curve using the VG model. The values of \(\theta_{s0}\), \(\theta_{r0}\) were calculated as 0.438 and 0.025, respectively, in this study. The values of \(a\), \(n\) parameters of the VG model were estimated using the least square method. Moreover, the range of the above parameters was considered according to the previous studies, where \(a\) was set within the range of 0-1 cm\(^{-1}\) and \(n\) was between 1 and 10 [15], [16].

3.3 Development of SWCC using Prediction Models
SWCC was developed using the models suggested by Arya & Paris [7], Aubertin et al. [11] and Wang et al. [12]. First, the curves were developed using the coefficient values recommended by the authors for granular soil.

According to the studies discussed in Section 2.2.4, it can be observed that the main reason for the deviations is that the model coefficients proposed by the authors rely on the soil type. Hence, recalculating the model parameters for the specific soil types by comparing the experimental data would reduce the deviations. Therefore, model parameters were recalculated for the soil type mentioned in this study using the least square method by comparing the experimental data, and the curves were developed accordingly. Figure 6
represents the summary of the methodology followed.

Figure 6 - Flowchart for the Research Methodology

4. Results & Discussion

4.1 Developing SWCC using Experimental Methods

The experimental results were obtained using WP4C in the range of 0 to 10^4 kPa, and these results were verified using the pressure plate and suction-moisture sensors. The 5 bar pressure plate produced results for matric suction in the range of 0 to 200kPa, whereas moisture-suction sensors covered the 7 to 365 kPa within four months. It was observed that some of the results obtained using WP4C have a deviation compared to those of the pressure plate and suction-moisture sensors. This could be due to the minor variation in the dry density of soil samples used in the WP4C because maintaining the dry density of the prepared soil samples at 1500 kg/m^3 is a difficult task as the height of the prepared samples was low. Accordingly, the experimental results obtained from all three instruments were used to develop SWCC for the selected soil type, and it is represented in Figure 7. Then the fitted SWCC was developed using the VG model, which also is shown in Figure 7. The curve equation of the VG model is represented by Equation 8.

\[ \theta = 0.025 + 0.375 \left[ \frac{1}{1 + (0.0845|h|)^{1.561}} \right]^{0.392} \quad \text{(8)} \]

where \( h \) is the suction matric head in cm. This soil type's corresponding \( \alpha \) and \( n \) values were derived as 0.0845 and 1.561, respectively.

Figure 7 - Developed SWCC using Experimental Data

4.2 Developing SWCC using Prediction Models

Figure 8 represents the SWCCs developed using the model parameters mentioned in Table 2.

Table 2 - Existing Values for Different Modal Parameters

<table>
<thead>
<tr>
<th>Model</th>
<th>Parameter</th>
<th>REC values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arya-Paris (1981)</td>
<td>( \alpha )</td>
<td>1.38</td>
</tr>
<tr>
<td>Aubertin et al. (2003)</td>
<td>( a_c )</td>
<td>0.01</td>
</tr>
<tr>
<td>Wang et al. (2017)</td>
<td>( C_1 )</td>
<td>1.07</td>
</tr>
<tr>
<td></td>
<td>( C_2 )</td>
<td>12.07</td>
</tr>
</tbody>
</table>

Figure 8 - Developed SWCC According to Prediction Models using Values Proposed by Authors for the Modal Parameters for the Granular Soils

It was observed that the curve developed using the Arya & Paris [7] model significantly differs from the experimentally developed curve. This
could be because the model is directly related to the particle size distribution; therefore, only a limited set of data can be used for the comparison which leads to unreliable predictions. However, the models developed by Aubertin et al. [11] and Wang et al. [12] can be used for a broader range of matric suction, allowing for a comparison that produces accurate predictions. The deviations obtained from prediction models are listed in Table 3.

### Table 3 - Comparison of Prediction Models with Values Proposed by Authors for Granular Soils

<table>
<thead>
<tr>
<th>Model</th>
<th>Parameter</th>
<th>REC values</th>
<th>SSR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arya-Paris (1981)</td>
<td>α</td>
<td>1.38</td>
<td>N/A</td>
</tr>
<tr>
<td>Aubertin et al. (2003)</td>
<td>αc</td>
<td>0.01</td>
<td>0.1605</td>
</tr>
<tr>
<td>Wang et al. (2017)</td>
<td>C1</td>
<td>1.07</td>
<td>0.0513</td>
</tr>
<tr>
<td></td>
<td>C2</td>
<td>12.07</td>
<td></td>
</tr>
</tbody>
</table>

This study was further extended by recalculating the model parameters of Aubertin et al. [11] and Wang et al. [12] models such that the SSR among the VG-fitted curve and the models became the least. As per the reasons mentioned above, Arya & Paris [7] model was excluded from this study.

Predictions of Aubertin et al. [11] deviated from the experimental results for matric suction values less than 3 kPa. The parametric values obtained for this soil type are listed in Table 4, including the deviations, and Figure 9 shows the SWCC developed for the new model parameters.

Table 4 - Derived Values and Deviations with the Experimental Results for Prediction Models

<table>
<thead>
<tr>
<th>Model</th>
<th>Derived values for this soil type</th>
<th>SSR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aubertin et al. (2003)</td>
<td>αc 0.0327</td>
<td>0.08</td>
</tr>
<tr>
<td>Wang et al. (2017)</td>
<td>C1 0.44</td>
<td>4.17 E-12</td>
</tr>
<tr>
<td></td>
<td>C2 6.77</td>
<td></td>
</tr>
</tbody>
</table>

5. Conclusions

Based on the verification obtained through the use of a pressure plate and a suction-moisture sensor, it is evident that the WP4C method employing the chilled mirror technique can yield prompt, dependable, and consistent results across a wider range of total suction values. However, it is crucial to exercise caution when handling samples in order to preserve the fundamental soil characteristics, such as dry density, due to the inherent challenges in collecting and preparing soil samples.

The experimentally derived curve, which was constructed utilizing Van Genuchten's (VG) model, yielded α and n values of 0.0845 and 1.561, respectively, for this specific soil type.

Upon comparing the experimental data, it was determined that the predictions provided by Aubertin et al. [11] and Wang et al. [12] models outperformed the Arya & Paris [7] model based on an analysis employing the least squares method. Consequently, the αc value for the Aubertin et al. [11] model was assessed as 0.0327, while the C1 and C2 values for the Wang et al. [12] model were found to be 0.44 and 6.77, respectively, for this soil type, effectively minimizing discrepancies between analytical predictions and experimental results.

The deviation between the Wang et al. [12] model and the experimentally fitted curve was calculated to be 4.17 E-12 for the proposed model parameters. As a result, the Wang et al. [12] model is deemed more suitable for generating Soil-Water Characteristic Curves.
analytically for this particular soil type, using the aforementioned model parameters.

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