

Estimation Criteria for Static Rock Mass Deformability Modulus for Rock-Socket Design in Metamorphic Rock Masses

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Abstract: This study investigates the most appropriate empirical criteria to estimate the static rock mass deformability modulus (E_m) in the design of rock-sockets in cast-in-situ bored pile construction. The in-situ E_m values are initially estimated through back analysis of static pile load test data. Secondly, the rock mass deformability estimated from back analysis (E_{mb}) are tested statistically against selected established empirical equations to determine whether the latter are appropriate for use in metamorphic rock terrain of Sri Lanka. It is found that the existing empirical criterion based on the square root of intact unconfined compressive strength (σ_c) derived from back analysis of pile load test results is appropriate for weak-poor rock masses. For strong-poor rocks, it is recommended to employ the equation based on σ_c , and in general the two equations generate lower and upper bound solutions. The equation based on intact deformability modulus (E_i) performs well in strong-excellent quality rock masses, while the equations based on E_i and rock quality designation (RQD) are found to be appropriate for weak-fair to excellent rock masses. Finally, a new set of equations appropriate for different rock mass types have been proposed through regression analysis along with appropriate design measures to be adopted.

Keywords: Rock mass deformability modulus, Intact deformability modulus, In-situ testing, Empirical method, Rock socket

1. Introduction

Reasonable estimation of rock mass deformability modulus (E_m) is essential when rock sockets are designed based on elastic theories. This requirement has a profound effect in strong crystalline rock masses compared to softer sedimentary formations as the elastic parameters are significantly higher in the former. Therefore, a significant impact is made by the parameter on the bearing capacity of rock sockets, especially in the skin friction component [1,2].

When considering the significance of the static rock mass deformability modulus E_m in rock socket design, it may be utilized to represent the linear elastic (no slip) as well as the non-linearinelastic (partial or full slip) phase of the rock socket deformation during the load application. The values representing these two phases can be separately employed to estimate the ultimate skin friction and the corresponding settlements pertaining to each phase.

However, the application of the E_m into real design practice in the design of rock sockets is not common. This is mainly due to the investment that is required for the

experimental evaluation of this parameter and even if such is made, the differences observed between the measured settlements and the

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theoretical settlement estimations made by adopting the experimentally evaluated E_m results.

It is reported that, the experimentally evaluated E_m values themselves (both intact and in-situ) show considerable discrepancies due to dependency on the test method [3] and the geological anisotropy [4].

Under these circumstances, the commonly adopted practice in the estimation of E_m among the practitioners in the piling industry is through the established empirical equations, though it is found to be conservative. Conversely, most of the empirical equations adopted in the rock socket designs are developed either from data bases established for comparatively weak rock masses or results obtained from different construction purposes such as tunnels, mines and dams.

Hence this study has been carried out to identify the most appropriate criteria in the estimation of E_m that suit stronger crystalline metamorphic rock mass conditions such as those found in Sri Lanka. Common empirical equations employed in the rock socket designs and their adoptability are initially investigated.

Secondly the investigated empirical equations are tested to determine whether they are appropriate for use in design of rock sockets in crystalline metamorphic rock. Finally, a new set of empirical equations with better

prediction capability are developed to estimate E_m for the design of rock sockets.

2. Estimation of E_m using Empirical Methods and Its use for Rock-Socket Design

2.1 Common Empirical Estimation Criteria for E_m in Rock Socket Design

Equations presented in Table 1 are commonly adopted due to their simplicity and data availability in rock socket designs. Nevertheless, apart from the equations presented in Table 1, there are equations which relate E_m with degree of weathering (WD), rock mass classification systems, Rock Mass Rating (RMR), Rock Mass Quality (Q), Geological Strength Index (GSI) and Rock Mass Index (RMi) and the degree of disturbance due to excavation (D) or as combinations of aforementioned parameters coupled with intact strength and deformability parameters. The presently available borehole log data, which is the main source of information, lack certain essential data needed for the assessment of the appropriateness of such equations for the purpose of this work. Hence empirical equations which depend only on most commonly available data, such as Rock Quality Designation (RQD) and the laboratory determined intact unconfined compressive strength (σ_c) or rock deformability modulus (E_i), have been tested to determine whether they are appropriate for use in rock socket design.

Table 1 - Commonly Employed Empirical Equations in the Estimation of E_m for Rock Socket Design

Empirical Equation	Equation Number	Author/s
$E_m = 0.5E_i$... (1)	Palmström and Singh [5]
$E_m = 215\sqrt{\sigma_c}$ (MPa)	... (2)	Rowe and Armitage [6]
$\log_{10} E_m/\sigma_c = 2.73 - 0.49 \log_{10} \sigma_c/P_a$... (3)	Prakoso [7]
$E_m = 0.2\sigma_c$ (GPa)	... (4)	Palmström and Singh [5]
$E_m/E_i = 0.0231 RQD - 1.32$... (5)	Coon and Merritt [8]
$E_m/E_i = RQD/350; RQD < 70$... (6a)	Bieniawski [9]
$E_m/E_i = 0.2 + [(RQD - 70)/37.5]; RQD > 70$... (6b)	
$E_m/E_i = J$, related to RQD	... (7)	Kulhawy and Goodman [10]
$RQD > 57\%$; Equation (5)	... (8a)	Gardner [11]
$RQD < 57\%$; $E_m/E_i = 0.15$... (8b)	
$E_m/E_i = 0.2 * 10^{(0.0186RQD - 1.91)}$ -Lower bound	... (9a)	Zhang and Einstein [3]
$E_m/E_i = 1.8 * 10^{(0.0186RQD - 1.91)}$ -Upper bound	... (9b)	
$E_m/E_i = 10^{(0.0186RQD - 1.91)}$ -Mean	... (9c)	

where, E_m - rock mass deformability modulus, E_i - intact deformability modulus, σ_c - intact unconfined compressive strength, RQD - rock quality designation, J - average joint spacing (mass factor)

Equation (1) has been based on the consideration of the scale effects of E_i on E_m in rock blocks of 2 m to 4 m equivalent specimen diameter and considering the scale-based relationship between the unconfined compressive strength (σ_c) of test samples and rock blocks (σ_{cm}). It is found to perform well in massive rocks with few or no joints [5]. Equation (4) is an extension of Equation (1), deduced for rocks with modulus ratio (MR) (the ratio between the intact deformability modulus and the unconfined compressive strength) of 400 [5]. Rowe and Armitage [6] have proposed Equation (2), based on the back analysis of a large number of load test results on piles socketed into weak rock [6]. They have proposed to apply a partial factor of 0.7 in pile socket design when adopting Equation (2) to accommodate the factor proposed by other literature related to rock socket design under serviceability limit state. Equation (3) is an advanced version of Equation (2) and Prakoso [7] has used the same data base of Rowe and Armitage [6], which mainly included pile load test data pertaining to weak rock masses such as mudstone, shale and sandstone. The basis for Equation (5) is through a data base of in-situ E_m obtained from relatively less fractured ($RQD > 60\%$) rock masses underlying dam sites of mainly granite gneiss and gneiss formations, as well as limestone and sandstone in some locations. Actually, Equation (5) is only applicable to rock masses with $RQD \geq 64\%$, else the elasticity modulus ratio (E_m/E_i) (which in some instances is defined as modulus reduction factor) becomes negative. Nevertheless, Coon and Merritt [8] recommend the equation to be used in small projects, where conducting in-situ tests is uneconomical. Moreover, the equation has been proven to perform well in the evaluation of the safety of existing rock founded concrete dam foundations against flood-overtopping [12]. Coon and Merritt [8] have proposed the modulus reduction factors in Table 2 to be adopted for different ranges of RQD . Equations (6a) and (6b) are based on the plot developed by Bieniawski [9] on a data base of in-situ E_m

corresponding to both weaker rock formations (siltstone, mudstone, sandstone, greywacke and shale) and stronger rock formations (massive granite gneiss, phyllite and dolerite). Based on the plot developed by Bieniawski [9], subsequent literature has presented and recommended these two equations for different rock applications and also for rock socket design. By considering the theoretical concepts that jointed rock can be modelled as an equivalent elastic continuum and that the deformability of a jointed rock mass is the result of the stiffness of the rock itself and the stiffness of the joints, Kulhawy and Goodman [10] had proposed Equation (7). Gardner [11] improved Equation (5) suggesting the form $E_m = \alpha_E E_i$ where α_E represents the coefficients given in Equation (5) to form Equation (8a) when $RQD > 57\%$ and Equation (8b) when $RQD < 57\%$, where it becomes 0.15. It is noteworthy to mention that these two equations have two main limitations as suggested by Zhang and Einstein [3]. Firstly, it does not sufficiently cover the case of $RQD < 60\%$, where an arbitrary modulus reduction factor value is proposed. Secondly when $RQD = 100\%$ it results in $E_m = E_i$, which is unrealistic and thus unsafe in design practice. This is because the case of $RQD = 100\%$ does not always imply that the rock is intact as there may be discontinuities in rock masses and thus the actual E_m may be less than E_i even though RQD is 100% [3]. In order to mitigate the deficiencies identified in the various previous relationships on E_m/E_i and RQD , Zhang and Einstein [3] have incorporated more data extracted from published literature into their data base with a greater range of RQD ($0 \leq RQD \leq 100\%$) and rock types (mudstone, siltstone, sandstone, shale, dolerite, granite, limestone, greywacke, gneiss, and granite gneiss). With the new data base, they have been able to develop Equation (9c) with two other equations which demarcate the lower bound Equation (9a) and upper bound Equation (9b) solutions for E_m/E_i against RQD relationships. Unlike in previous instances, their relationships are found to be nonlinear, but with a better coefficient of regression.

Table 2 - Modulus Reduction Factors for Different RQD Levels [8]

RQD (%)	E_m/E_i
00-25	<0.20
25-50	<0.20
50-75	0.20-0.50
75-90	0.50-0.80
90-100	0.80-1.00

2.2 Appropriate Criteria for Rock Socket Design

Pells and Turner [1] have pioneered the dominant use of elastic parameters of rock in the design of rock sockets by producing settlement estimation charts for different socket types and geometries, in which E_m plays an integral role. They proposed



elastic solutions for the settlement of shear sockets, flexible or rigid type end-bearing sockets (considering a loaded circular area at the base of a shaft), and for complete sockets (which offer resistance by skin friction over the socket wall as well as end bearing at the base of shaft). They further suggest modified solutions for recessed shafts due to embedment, while reporting that their solutions for the settlement of end-bearing bases better represent the case of flexible footings (hard rock) compared to that of rigid footings (soft rock). Moreover, Pells and Turner [1] have suggested adopting the settlement reduction factors yielded by solutions for rigid footings when $E_{pile}/E_m > 50$ and otherwise through solutions proposed for flexible footings. Pells and Turner [1] also recommend a criterion to estimate E_m through back analysis of pile load tests to verify the accuracy of the design solution. Coupling the field and laboratory experimental results with the elastic solution of Pells and Turner [1], Williams and Pells [13] introduced a side resistance reduction factor for rock sockets for the estimation of ultimate rock socket skin friction for a particular rock zone. The side resistance reduction factor (β) represents the reduction of lateral confinement which is experimentally found to be proportionately related to J (mass factor) = E_m/E_i , given in Equation (7) and has also been directly related to the joint spacing and hence RQD [10]. Williams and Pells [13] demonstrated that in-situ E_m can be estimated through back analysis of field load test results on piles to an accuracy of within 2 times (upper bound) and 0.5 times (lower bound) of the E_m estimated from in-situ pressure-meter test results. Comprehensive elastic solutions have been proposed by Kulhawy and Carter [14] in the design of rock sockets and they concluded that elastic solutions for shear sockets are more accurate for larger E_{pile}/E_m ratios (soft rocks), while solutions for complete sockets are found to be conservative but satisfactory for most of the design cases. Similar elastic solutions, but extending to displacements up to full-slip conditions, have been proposed by Rowe and Armitage [15], in which all the design solutions are graphically presented in the form of E_{pile}/E_m ratio, making the deformability modulus an integral component. They recommend to adopt their own empirical Equation (2), arrived through back analysis of pile load test results, with a partial safety factor of 0.70 to compensate for any uncertainties.

Similar to the estimation of E_m , a substantial number of empirical formulae are available for the estimation of bearing capacity in the design of rock sockets. Though the basis for most of the design solutions are aforementioned elastic solutions, the time and cost involved in obtaining the required parameters like E_m have caused the practitioners to resort to cheaper empirical means with reasonable accuracy to obtain such parameters. The best example is the popularly adopted rock socket skin friction estimation criteria proposed by Williams and Pells [13]. Though it has originated through a theoretical-experimental background, the way it is adopted in the design [16] involves a high degree of empiricism. As an extended version to Table 2 [8], O'Neill et al. [17] proposed modulus reduction factors presented in Table 3 by considering the joint characteristics, for the design of rock sockets.

Table 3 - Modulus Reduction Factors Based on RQD Levels [17]

RQD (%)	E_m/E_i	
	Closed Joints	Open Joints
100	1.00	0.60
70	0.70	0.10
50	0.15	0.10
20	0.05	0.05

Moreover, Load and Resistance Factor Design for Bridge Design Specifications (LRFDBDS) [18] proposes to adopt the least of the two values obtained for E_i , from intact core sample test and from Equation (10), for the design of rock sockets.

$$E_m = E_i \left[\frac{E_m}{E_i} \right]_t \quad \dots (10)$$

where,

E_i - Obtained from intact core sample test, and

$\left[\frac{E_m}{E_i} \right]_t$ - Obtained from Table 3.

When laboratory estimated E_i data are not available, Equation (11) proposed by Hoek and Diederichs [19] can be objectively adopted.

$$E_i = MR * \sigma_c \quad \dots (11)$$

where,

MR - modulus ratio, representing the ratio between the intact deformability modulus and the unconfined compressive strength, which is generally found to be constant (at least in a range) for a particular rock type and a texture [5,19].

In combination with Equations (7) and (11), BS 8004:1986 [20] proposes Equation (12) to estimate E_m .

$$E_m = J * MR * \sigma_c \quad \dots (12)$$

This approach has also been recommended by Williams and Pells [13] and is abundantly used as an indirect empirical approach in the rock socket design [16].

3. Methodology

The following methodology was adopted in this work, which resembles the processes adopted in a number of similar previous studies.

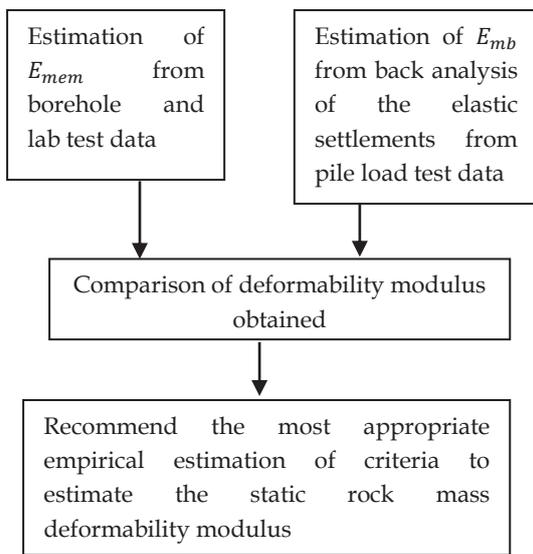


Figure 1 - Conceptual Study Framework (where, E_{mem} - empirically evaluated rock mass deformability modulus; E_{mb} - rock mass deformability modulus evaluated from back analysis of pile load test data)

As depicted in Figure 1, the rock mass deformability modulus (E_{mem}) is initially estimated using different empirical formulae reported in Table 1.

Due to the non-availability of in-situ E_m data, the technique of back analysis of pile load test data, which had previously been used by many research studies in pile foundation designs, has been adopted to estimate the in-situ E_m in this work. Estimation of in-situ E_m was carried out in two stages, viz., first to estimate the elastic settlement component of the rock socket and then to evaluate E_m corresponding to the estimated elastic settlement.

The estimation of elastic settlements corresponding to rock socket section of the pile

was carried out through the load-displacement curve interpretation techniques. The availability of fully instrumented static load test data in the Sri Lankan context is very rare due to its high cost, while general static load test data are available to some extent. Out of such, availability of complete sets of data to the authors was further limited. The limited available data on instrumented load tests were not fully adoptable for this work as most of the rock sockets had not even reached the ultimate elastic limit due to, either the majority of the skin friction resistance had already been taken through the very thick soil profile or else the load had been taken by the lengthy rock sockets. Therefore, only 74 numbers of conventional static load test data were employed with the generous courtesy of the respective project owners and piling contractors.

Unlike in fully instrumented load tests, estimation of the elastic deformations corresponding to the rock sockets through conventional non-instrumented static load test data was cumbersome. Nevertheless, it was carried out in a three-staged process. Initially, the total elastic settlement of the pile was estimated using the procedure described by Thilakasiri et al. [21] as depicted in Figure 2. This is by drawing a tangent to the initial straight section of the load settlement curve and obtaining the maximum settlement of the pile corresponding to the farthest point of tangency on the load settlement curve. In the second stage, ultimate skin friction force (U_s) generated along the shaft by the surrounding soils was estimated with Equation (13) under the following assumptions: (i) the vertical stress distribution along the shaft follows the critical depth concept [22]; and (ii) majority of the soil layers along the pile shaft have been mobilised to their respective ultimate skin friction levels by the time the rock socket reached its maximum elastic limit. In the third stage, the resultant elastic shortening of the pile due to the downward total applied force (P_T) and the ultimate frictional resistance acting upwards (U_s) generated by the surrounding soils were estimated using the technique proposed by Fleming [23] as elaborated in Figure 3 with Equation (14). Finally, the obtained resultant deformation was eliminated from the initially estimated total settlement to determine the elastic deformation corresponding to the rock socket section of the pile. The procedure adopted for non-instrumented load tests has



been tested for a limited number of available instrumented load tests as well, which produced reasonably accurate results (within 15%) on the elastic deformation of the rock socket as depicted in Table 4.

$$\phi^o = \phi' - 3 \quad \dots (13)$$

where,
 ϕ' - Effective angle of internal friction of soil strata prior to the installation of a bored pile
 ϕ^o - Modified effective angle of internal friction of soil strata subsequent to the installation of a bored pile.

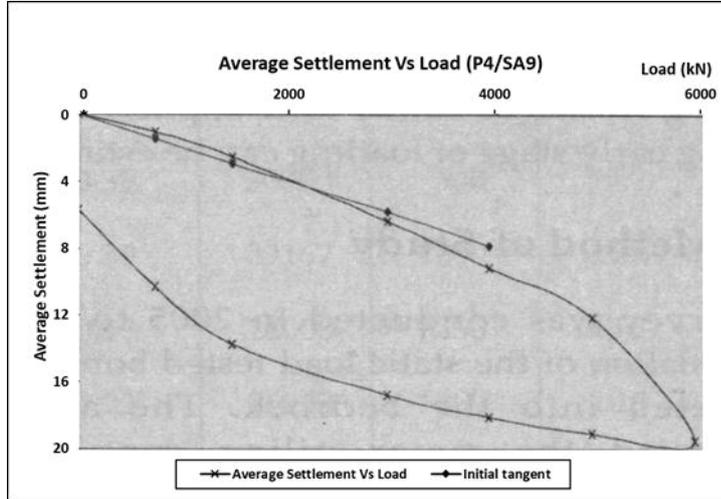


Figure 2 - Establishment of Initial Elastic Deformation Range from Load- Settlement Curve [21]

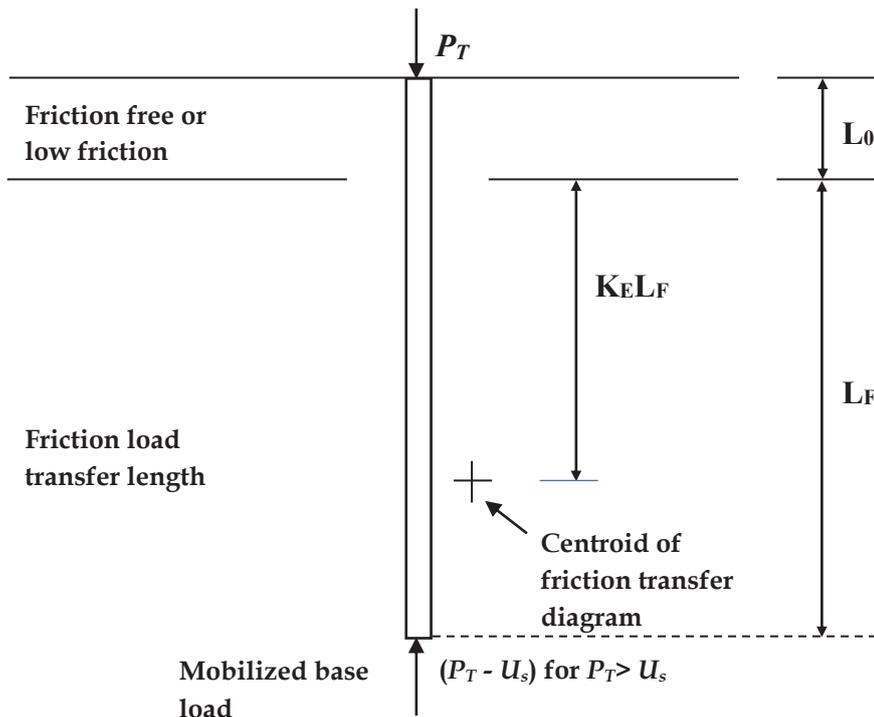


Figure 3 - Simplified Method of Evaluating the Elastic Shortening of the Shaft [23]

Table 4 - Comparison of Results Obtained from Instrumented Load Test and Flemming [23]

Location	Instrumented Load Test		Method adopted for non-instrumented Load Test in this study	
	Ultimate elastic stress on rock socket (kPa)	Corresponding elastic displacement of rock socket (mm)	Ultimate elastic stress on rock socket (kPa)	Corresponding elastic displacement of rock socket (mm)
Colombo 1	63.38	3.10	55.01	2.76

The elastic shortening of the pile (Δ_{SE}) is obtained through the parameters depicted in Figure 3.

$$\Delta_{SE} = \frac{4}{\pi} \frac{1}{D_s^2 E_c} [P_T(L_0 + L_F) - L_F U_s (1 - K_E)] \dots (14)$$

where, D_s is the diameter of the pile shaft and K_E is the effective length factor representing the ratio between the depth to the centroid of friction transfer diagram (which is the diagrammatic representation of the frictional force enforced by different soil strata along the pile shaft) from the starting point of the friction load transfer section and the friction load transfer length (L_F). In case of piles of non-circular sections, equivalent diameters can be used [23].

The theoretical estimation of elastic settlements of the rock sockets concerned was carried out through three different widely used rock socket design criteria, which encompass linear elastic deformations, viz., Pells and Turner [1], Kulhawy and Carter [14] and Rowe and Armitage [15] criteria, respectively, given under Table 5. The respective elastic solutions are given in graphical form in the respective publications to obtain the settlement influence factor (I_ρ) for different socket geometries and elastic modulus ratios (E_p/E_m). With the established I_ρ , the elastic settlements evaluated for rock sockets through pile load test data discussed earlier were replaced with the corresponding elastic settlements (ρ) in Equation (15) to obtain the back analysed deformability modulus of rock mass (E_{mb}):

$$E_{mb} = \left[\frac{F}{r \rho} \right] I_\rho \dots (15)$$

where, r - radius of the pile/socket, F - effective compressive load of the pile at the top of the rock socket, which was taken as

$$F = P_T - U_s \dots (16)$$

In order to use above elastic settlement charts, an initial estimate of E_m is essential. For this purpose, the widely accepted criterion of Equation (11) was adopted to initially estimate the E_i (since the availability of the parameter for projects in Sri Lanka is very sparse) with appropriate MR (Note: Findings of the authors for metamorphic rocks in Sri Lanka yield an average value of 412). Then, with the corresponding J related to RQD and from Equation (7), the initial estimate of E_m was evaluated. Moreover, isotropic condition on

E_m was assumed to prevail for rock-socket wall (E_m) and end bearing rock (E_b) and hence charts pertaining to $E_b/E_m = 1$ were only considered in this analysis. The remaining factors, such as Poisson's ratio (ν) of rock and pile were assumed as 0.25 (as described in the respective publications). As depicted in Table 5, theoretical estimations can be carried out for both shear as well as complete socket conditions using the first and the second methods, while the third method facilitates only complete socket condition. Back analysis has been carried out for all the five design criteria and the corresponding deformability moduli (E_{mb}) have been obtained.

Table 5 - Analysis Method

Rock socket design criteria	Considered type of the socket condition
Pells and Turner [1]	Shear and Complete sockets
Kulhawy and Carter [14]	Shear and Complete sockets
Rowe and Armitage [15]	Complete socket

The empirically evaluated rock mass deformability modulus (E_{mem}) obtained using the Equations (1) to (9c) were compared against the deformability modulus obtained through back analysis (E_{mb}) to check the appropriateness of each equation to estimate the static rock mass deformability modulus for rock socket designs in hard metamorphic rock such as those existing in Sri Lanka. This is by performing a Root Mean Square Error (RMSE) analysis. This technique has previously been used by a number of researchers for the same purpose. The smaller the error generated, the better the predictive capacity of the particular equation. Finally, by performing linear as well as non-linear regression analyses using the Statistical Package for the Social Sciences (SPSS) software, a novel set of empirical equations are developed to obtain the E_m value for metamorphic rock socket designs.

4. Results and Discussion

4.1 Results of Modulus of Deformability from Back Analysis (E_{mb})

Table A1 comprehensively presents the data extracted, analysed and evaluated in the estimation of E_{mb} under Pells and Turner [1] criteria for both shear and complete socket conditions, for a collection of cases in Sri Lanka. As mentioned earlier, it has been noticed during the load-settlement curve



interpretation that, though all have reached their maximum elastic deformation, most of the piles have not reached even the partial-slip condition. The maximum linear elastic phase of the socket is found to occur at a total pile displacement of around 0.004% to 1.0% of the pile diameter, which is comparable with results reported elsewhere [24].

Furthermore, Table A2 reports the E_{mb} values estimated for each location under all the five design criteria (two for shear sockets and three for complete sockets) mentioned in Table 5, which include both shear and complete sockets. Accordingly, the least values are generated for complete sockets by both Pells and Turner [1] and Kulhawy and Carter [14] methods, while the latter generates the least. Though Rowe and Armitage [15] method facilitates the secondary slip behaviour of rock sockets, it generates significantly high values (about 73% higher than Kulhawy and Carter [14] method), even after adopting a reduction factor of 0.7 as suggested by the authors. Table 6 summarises the results for Pells and Turner [1] and Kulhawy and Carter [14] methods. Since there is no proven evidence to suggest that any of the rock sockets concerned belonged to the shear socket category, the analysis was continued further only for the results obtained for complete rock sockets.

4.2 Appropriateness of Existing Empirical Equations in the Design of Rock-Sockets

The set of values obtained for E_{mb} for complete sockets (which is generally the lesser) has been adopted to evaluate the RMSE against the values estimated through empirical equations E_{mem} . The results are reported in Table 7 and as it depicts, Equations (6) to (9c) produce the least RMSE values under all the design criteria. Out of the three main design criteria, Pells and Turner [1] and Kulhawy and Carter [14] methods produce reasonably lower results, whilst Rowe and Armitage [15] method yields considerably higher deviation. Equation (9c) performs best among other equations, followed by (6) for Kulhawy and Carter [14] method, while (6) and (7) perform well for Pells and Turner [1] method. Out of the

Equations (1) to (4), which solely depend on intact parameters, Equations (2) and (3) perform best under Kulhawy and Carter [14] method and the performance of both equations are better when compared to the remaining two in Pells and Turner [1] method. Though RMSE provides the level of the overall predictability of respective empirical equations, the same has been graphically presented in Figures 4 (a) to 4 (c) to investigate their sectorial behaviour for the results obtained through Kulhawy and Carter [14] method. As Figure 4 (a) depicts, Equation (1) over predicts the rock mass deformability value across all the intact deformability levels. However careful observation shows that high-end E_m values corresponding to high strength ($\sigma_c > 100\text{MPa}$)-fair ($RQD > 50\%$) rocks reasonably match with Equation (1). Moreover, majority of the actual deformability values (E_{mb}) lies in the proximity and below the Heuze [25] lower bound solution of 20% of the intact value. Considering the volumes involved in the rock socket deformation compared to the test volume and the discontinuity intensity, the results obtained are found to be appreciably consistent with the Heuze [25] findings. Figure 4 (b) suggests that Equations (2) or (3) performs best for medium to low strength rocks ($\sigma_c < 100\text{MPa}$), while the performance of both are appreciable for low quality rocks across all the strength ranges. Nevertheless, Equations (2) or (3) and Equation (4) can be accepted as an envelope which provides lower and upper bound solutions, respectively. It is obvious from Figure 4(c) that overall performance of Equations (6) to (9c) observed in RMSE is not graphically evidenced, most probably due to lack of high-strength high- RQD data. Nevertheless, Equation (9c), which possesses the least RMSE, fits with appreciable amount of actual rock mass deformability values, and Equation (9a) and Equation (9b) can be found to produce an upper bound solution for most of the cases, which agrees with the findings of Zhang and Einstein [3]. Table 8 compares the modulus reduction factors proposed by Coon and Merritt [8] and O'Neill et al. [17] with the observations made in this study.

Table 6 - Summary of Results Obtained from Different Back Analysis Methods

Rock socket design criteria	Difference between results obtained from complete and shear sockets		Overall difference between results obtained for complete sockets from two methods	
	Range%	Mean%	Range%	Mean%
Pells and Turner [1]	2.78-30.00	11.21	0.00-27.60	12.37
Kulhawy and Carter [14]	0.21-49.15	28.61		

Table 7 - The values of Root Mean Square Errors (RMSE) Estimated from the Empirical Equations (E_{mem}) against the Rock Mass Modulus of Deformability Values from Back Analysis (E_{mb})

Empirical Equation	RMSE		
	Pells and Turner [1]	Kulhawy and Carter [14]	Rowe and Armitage [15]
(1) Palmström and Singh [5]	12.85	13.01	12.48
(2) Rowe and Armitage [6]	10.01	8.91	14.57
(3) Prakoso [7]	10.15	9.05	14.73
(4) Palmström and Singh [5]	12.45	12.58	12.23
(5) Coon and Merritt [8]	10.97	11.05	18.15
(6) Bieniawski [9]	5.30	5.10	9.16
(7) Kulhawy and Goodman [10]	5.22	5.35	8.75
(8) Gardner [11]	5.51	5.57	8.98
(9.a) Zhang and Einstein [3]	9.73	8.65	14.41
(9.b) Zhang and Einstein [3]	6.56	7.18	7.66
(9.c) Zhang and Einstein [3]	5.51	4.97	9.73

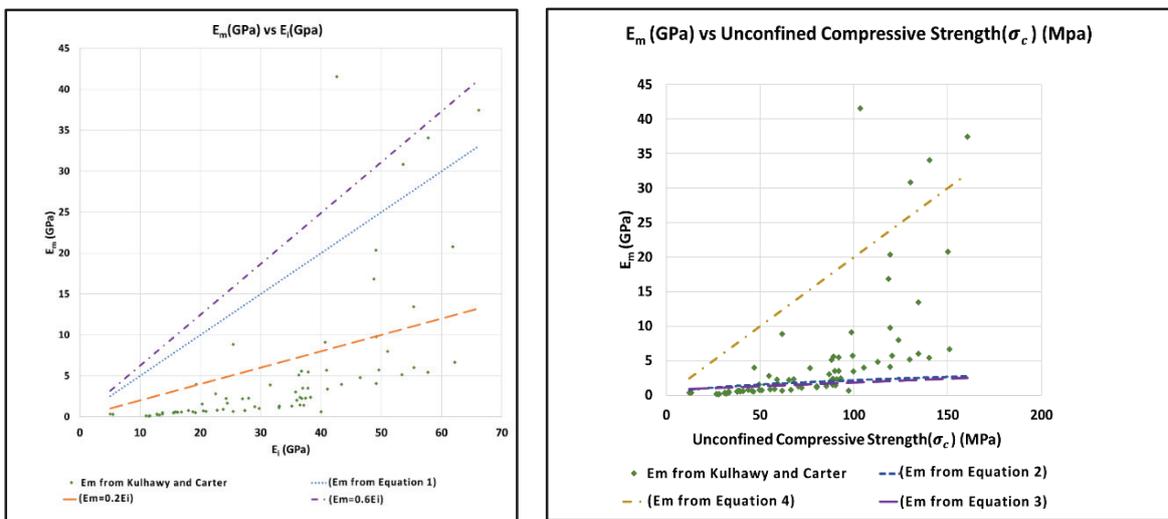


Figure 4(a) - Actual behaviour of (E_m) against (E_i) with existing empirical relationships **Figure 4(b) - Actual Behaviour of (E_m) against (σ_c) with Existing Empirical Relationships**

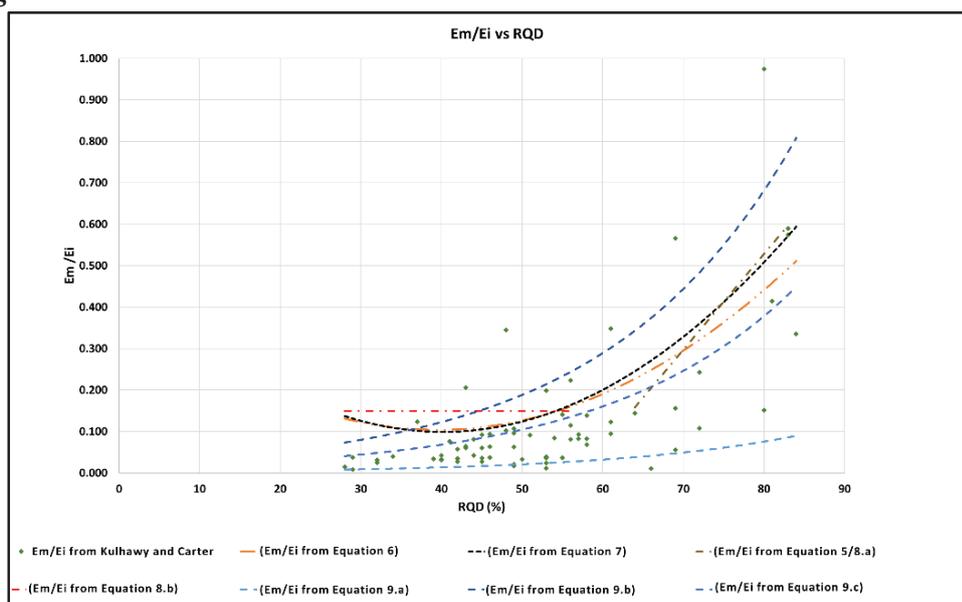


Figure 4(c) - Behaviour of Deformability Modulus Ratio (E_m/E_i) against (RQD) with Existing Empirical Relationships



Table 8 – Comparison of Modulus Reduction Factors Proposed by [8] and [17] with the Present Results

RQD (%)	E_m/E_i			
	Coon and Merritt [8]	O’Neill et al. [17]	Current study	
			Range	No. of Data
<25	<0.20	<0.05	-	00
25-50	<0.20	0.05-0.15	0.01-0.35	38
50-75	0.20-0.50	0.15-0.70	0.01*-0.57	30
75-100	0.50-1.00	0.70-1.00	0.15*-0.97	06

*Corresponds to low unconfined compressive strength ($\sigma_c < 100\text{MPa}$) results for $E_m/E_i < 0.2$

As the Table 8 depicts, though a reasonable agreement prevails with previously suggested values, distinct ranges of modulus reduction factors are not evident, especially for low strength rock masses of $RQD > 50\%$.

4.3 Results on Regression Analysis

With the outcomes reached from section 4.2, a new set of equations have been tested for E_m , against the intact deformability (E_i),

Table 9 – Results of the Regression Analysis

Relationship	Type of equation	Equation	Coefficient of regression (r)
E_m Vs UCS (σ_c)	Linear	$E_m = 0.145\sigma_c - 6.197... (17)$	0.623
	Log	$E_m = 8.064 \ln \sigma_c - 28.910... (18)$	0.525
	Exp	$E_m = 0.148e^{0.033 \times \sigma_c} ... (19)$	0.863
	Power	$E_m = 0.0002\sigma_c^{2.128} ... (20)$	0.841
E_m Vs E_i	Linear	$E_m = 0.353E_i - 6.197... (21)$	0.623
	Log	$E_m = 8.064 \ln E_i - 21.760... (22)$	0.525
	Exp	$E_m = 0.148e^{0.081 \times E_i} ... (23)$	0.863
	Power	$E_m = 0.002E_i^{2.128} ... (24)$	0.841
E_m/E_i Vs RQD	Linear	$E_m/E_i = 0.008RQD - 0.289... (25)$	0.657
	Log	$E_m/E_i = 0.376 \ln RQD - 1.353... (26)$	0.598
	Exp	$E_m/E_i = 0.005e^{0.050 \times RQD} ... (27)$	0.673
	Power	$E_m/E_i = 3.683 \times 10^{-6} RQD^{2.517} ... (28)$	0.654

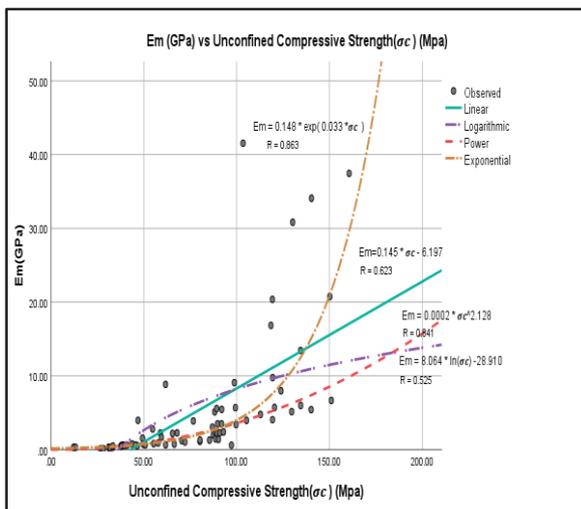


Figure 5(a) - Relationship between Unconfined Compressive Strength and Deformability Modulus

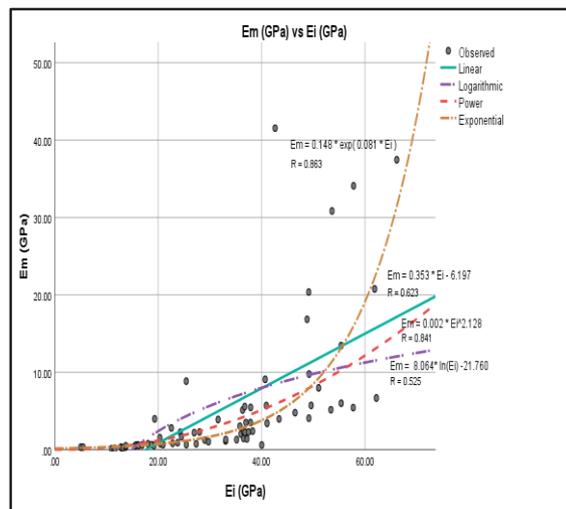


Figure 5(b) - Relationship between Intact Deformability Modulus and Deformability Modulus

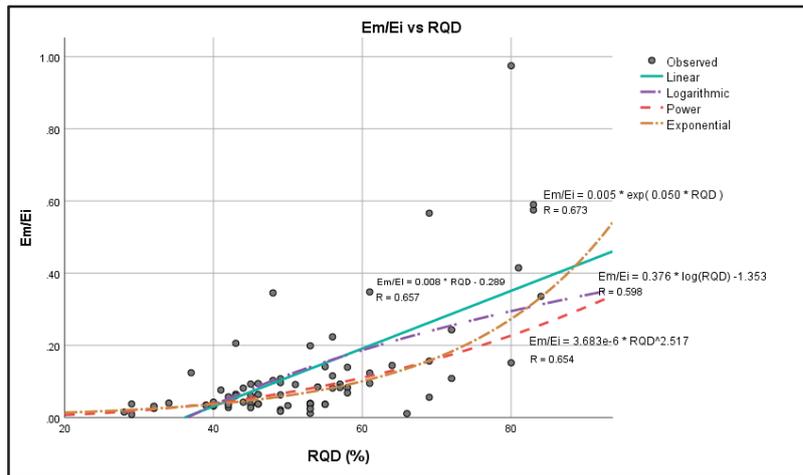


Figure 5(c) - Relationship Between Rock Quality Designation and Deformability Modulus Ratio

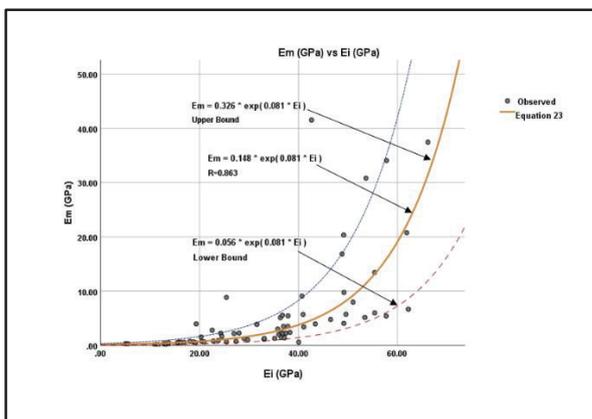


Figure 6(a) - Lower and Upper bound Solutions for Exponential Relationship between Unconfined Compressive Strength and Deformability Modulus

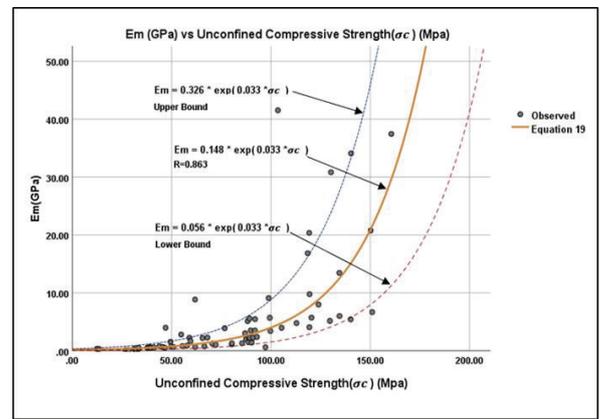


Figure 6(b) - Lower and Upper bound Solutions for Exponential Relationship between Intact Deformability Modulus and Deformability Modulus

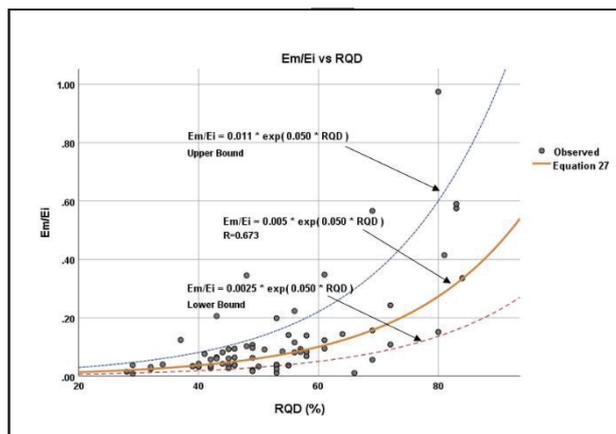


Figure 6 (c) - Lower and Upper bound Solutions for Exponential Relationship between Rock Quality Designation and Deformability Modulus Ratio

unconfined compressive strength, UCS (σ_c) and RQD through a regression analysis carried out on results obtained from Kulhawy and Carter [14] method. Table 9 summarises the results, while Figures 5 (a) to 5 (c) depict their respective graphical behaviour. Though Table 9 reports considerably higher regression (r)

values for relationships with σ_c , Figure 5 (a) suggests that Equation (19) performs best in all strength categories, while both Equations (19) and (20) perform equally well in medium to low strength rocks ($\sigma_c < 100$ MPa). The relationships with E_i , depicted in Figure 5 (b), are almost similar to the relationships with



σ_c in Figure 5 (a). Such can be expected, since E_i is estimated in this work by simple multiplication of σ_c with a constant MR . Surprisingly, the relationships with RQD generate weaker r values. However, as Figure 5 (c) suggests, Equations (27) and (28) perform reasonably well for all the RQD ranges.

Generally, the exponential and power type equations perform exceptionally well with r greater than 0.650, whilst the linear type equations also perform well with all the three investigated parameters with r greater than 0.600 in all three cases, which is quite acceptable considering the previous findings elsewhere.

In the practical application of the exponential relationships, two upper and lower bound solutions are proposed for Equations (19), (23) and (27), as depicted in Figures 6 (a) to 6 (c). Accordingly, all three equations generate upper bound solutions of around 220% and lower bound solutions of around 38% to 50% of the E_m estimated from the proposed exponential relationships. Interestingly, these upper and lower bound solutions match reasonably with the solutions proposed by Zhang and Einstein [3], which produce 180% and 20% of the E_m as upper and lower bound solutions, respectively. Also, the above bounds proposed by this work are analogous with the findings of Williams and Pells [13], in which E_m obtained through back analysis varied within 200% (upper bound) and 50% (lower bound) of the E_m obtained from in-situ tests.

5. Conclusions and Recommendations

5.1 Conclusions

The significant outcomes of this study can be broadly classified into the appropriateness of established equations and the practical application of the newly developed equations in the estimation of rock mass deformability modulus (E_m), in crystalline metamorphic rocks. Based on the outcomes of the study, following specific conclusions can be made, which are thus valid for metamorphic rock masses.

1. The values estimated for maximum deformation in the linear elastic range and the corresponding stress on rock socket for non-instrumented load tests is within acceptable range when compared with the results of instrumented load test (though very limited). Moreover, the elastic

deformations are comparable with the results of the previous data bases elsewhere. The difference between the E_m estimated from Pells and Turner [1] and Kulhawy and Carter [14] methods are within acceptable range (latter produced lower bound results), while Rowe and Armitage [15] produce significantly high results and the outcomes are consistent with the results reported by Lacy and Look [24]. Figure 4 (a) demonstrates that results obtained for E_m are consistent with the E_m to E_i relationship proposed by Heuze [25], at least conservative towards lower limit. Hence, considering all the above facts, it can be concluded that results obtained are acceptable.

2. Though the overall performance of Equation (1) is found to be poor, it is found to produce reasonably acceptable results for high strength ($\sigma_c > 100$ MPa)-fair ($RQD > 50\%$) rocks and the results are consistent with the comments made by the originators, Palmström and Singh [5].
3. Apart from very few data points, Equation (2) or Equation (3), and Equation (4) create an envelope of E_m , in which Equations (2) and (3) act as lower bound while Equation (4) as upper bound solutions. Equation (2) or Equation (3) performs exceptionally well for medium to low strength ($\sigma_c < 100$ MPa) rock masses, while Equation (4) performs well for high strength ($\sigma_c > 100$ MPa) rocks, which is consistent with the comments made by Palmström and Singh [5].
4. The combined effects of E_i and RQD are better represented by Equation (9c), while the Equations (6), (7) and (8) also produce reasonable results.
5. The developed Equation (19) performs reasonably well for all strength categories, while both Equations (19) and (20) perform equally well in medium to low strength rocks ($\sigma_c < 100$ MPa). Similar behaviour is observed for new Equations (23) and (24).
6. The effects of discontinuities are better represented by the new Equations (27) and (28) and these also are found to perform reasonably well for all RQD levels.

5.2 Recommendations

In the estimation of E_m in the design of rock sockets, following criteria can be recommended for different categories of metamorphic rock masses.

1. As a very crude measure, the following modulus reduction factors can be recommended to be adopted for higher intact rock strength magnitudes, ($\sigma_c > 100$ MPa).

RQD (%)	E_m/E_i
25-50	0.01-0.15
50-75	0.15-0.50
75-100	0.50-0.97

2. For medium to low intact rock strength magnitudes ($\sigma_c < 100$ MPa) and Poor to Very poor quality ($RQD < 50\%$), considering the fact that intact properties govern the behaviour of E_m in poor quality rock masses [26], either the previously established Equations (2) or (3) or else the newly established Equations (19) and (20) shall be adopted.
3. For high intact rock strength magnitudes ($\sigma_c > 100$ MPa) and Poor to Very poor quality ($RQD < 50\%$), either the previously established Equation (4) or else the newly established Equation (19) shall be adopted.
4. For high strength magnitudes ($\sigma_c > 100$ MPa) and better than Fair quality ($RQD > 50\%$), either the previously established Equation (1) (especially when $RQD > 90\%$) or else the newly established Equations (27) and (28) shall be adopted. This is by considering the fact that E_m of better-quality rock masses is controlled by the geological discontinuities [26].
5. For medium to low intact rock strength magnitudes ($\sigma_c < 100$ MPa) and better than Fair quality ($RQD > 50\%$), either the previously established Equation (9c) (which may generally provide an upper bound solution) or else the newly established Equations (27) and (28) shall be adopted.
6. In the adoption of Equations (19), (23) and (27) in the above rock mass conditions, appropriately factored E_m (ranges between $0.38 E_m$ lower-bound to $2.2 E_m$ upper-bound) shall be adopted in rock socket

designs, considering the other quality parameters of the particular rock masses to avoid unsafe or overdesign scenarios.

7. Though the adoptability of the investigated empirical equations has been tested only for Kulhawy and Carter [14] method, as Table 7 depicts, the applicability is found to be reasonably valid for Pells and Turner [1] method as well. Hence the above recommendations can be adopted for either of the methods during the rock socket design.
8. As Table 6 suggests, the E_m estimated for a shear socket yields a value 30% (maximum) higher than for a complete socket with Pells and Turner [1] method and it is as much as 50% when using Kulhawy and Carter [14] method. Conversely, E_m , estimated from the recommended criteria in this work (which is based on the complete socket results) can be conveniently adopted in shear socket designs with a multiplication factor equivalent to the reciprocal of the maximum difference found with the two different design criteria. That is $0.77 E_m$ and $0.66 E_m$, respectively, for Pells and Turner [1] and Kulhawy and Carter [14] methods. With such a design criterion, designers can effectively shorten the rock socket lengths in rock masses with possible highly fractured - low strength toe areas.
9. Careful observation of data suggests that the database is more biased to poor quality weaker rock formations. Hence it is recommended to further refine the newly proposed equations for high strength-good quality rock masses.
10. In order to further improve the findings, it is highly recommended to initiate similar kind of studies from the inception of the borehole investigations, from which all the parameters relevant to joints as well as weathering can be accurately obtained. Secondly, rather than depending on indirect transformations, the E_i values shall be experimentally determined with appropriate post treatments on results. In this work, neither in-situ test data (due to non-availability) nor a considerable number of instrumented load test results have been considered. However, results of these two stages are essential in such a



study and these will need to be amalgamated in such a comprehensive study in the future, which will also facilitate the adoptability of rock mass related parameters such as *RMR*.

Acknowledgement

Authors wish to firstly acknowledge the assistance given by the respective project owners and secondly the corresponding project consultants and the pile contractors in providing the necessary geotechnical and pile test data. Authors also would like to especially thank the Librarian and the staff of the Library of The Open University of Sri Lanka for their excellent and timely support given in finding the relevant literature. Special thanks go to Dr. K.D.N Kumari of Department of Mathematics and Philosophy of Engineering of Open University of Sri Lanka for her assistance given in data analysis.

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Table A1 - Results on the Estimation of Rock Mass Deformability Modulus through Back Analysis (E_{mb}) with Pells and Turner [1] criteria

Location	Rock type	RQD	UCS (MPa)	Initial Estimate E_i (GPa)	Mass Factor (f)	Initial Estimate E_m (GPa)	Maximum applied load on socket head in initial range, S	Maximum settlement in initial elastic range, S	Length of pile, L_1 (m)	Effective pile length factor, K_f	Diameter of pile shaft, B (mm)	Elastic Modulus of concrete, E_c (GPa)	Elastic shortening of pile, s (mm)	Settlement in rock socket (mm)	Depth of pile socket (mm) (D)	D/B	Stiffness ratio, (K)	2D/B	$f_{p(Concrete)}$	$f_{p(Rubber)}$	Radius of loaded area, a (m)	Elastic Modulus of rock mass (back analysis- Shear) E_{mb} (GPa)	Elastic Modulus of rock mass (back analysis- Complete) E_{mbc} (GPa)
Colombo 01	Moderately weathered Biotite Gneiss	54	86.91	35.81	0.160	5.73	1200	0.60	24.78	0.47	1200	30.00	0.41	0.19	1000	0.8	5.24	1.67	0.318	0.385	0.600	3.88	3.29
Colombo 01	Moderately weathered Biotite Gneiss	45	49.27	20.38	0.156	2.78	1200	0.60	24.78	0.47	1200	30.00	0.41	0.19	1000	0.8	5.24	1.67	0.318	0.385	0.600	3.88	3.29
Colombo 08	Moderately weathered Biotite Gneiss	41	72.15	29.76	0.164	5.67	1200	0.60	24.78	0.47	1200	30.00	0.41	0.19	1000	0.8	5.24	1.67	0.318	0.385	0.600	3.88	3.29
Alawwa	Slightly weathered Biotite Gneiss	56	98.27	40.63	0.164	6.57	1200	0.60	24.78	0.47	1200	30.00	0.41	0.19	1000	0.8	5.24	1.67	0.318	0.385	0.600	3.88	3.29
Alawwa	Moderately weathered Biotite Gneiss	28	97.17	40.03	0.075	3.00	600	0.30	12.39	0.47	1200	30.00	0.41	0.19	1550	1.3	8.66	2.58	0.320	0.260	0.600	0.83	0.70
Colombo 01	Moderately weathered Biotite Gneiss	57	59.91	24.27	0.170	4.13	3000	3.35	30.10	0.47	900	26.00	2.56	0.79	1550	1.1	6.30	2.22	0.320	0.300	0.450	2.72	2.52
Colombo 01	Moderately weathered Biotite Gneiss	53	80.27	33.07	0.150	4.96	1000	0.96	20.34	0.47	900	30.00	0.50	0.35	1000	1.1	6.05	2.22	0.315	0.280	0.300	1.52	1.45
Colombo 01	Moderately weathered Biotite Gneiss	55	85.37	35.17	0.162	5.70	500	1.05	21.89	0.47	600	26.00	0.70	0.35	1000	1.7	4.56	3.33	0.325	0.280	0.300	1.55	1.33
Colombo 01	Moderately weathered Biotite Gneiss	46	12.21	5.03	0.128	0.64	450	1.28	18.56	0.47	600	30.00	0.46	0.82	1000	1.7	46.59	3.33	0.230	0.200	0.300	0.42	0.37
Colombo 01	Moderately weathered Biotite Gneiss	56	120.37	49.59	0.168	8.33	1000	0.30	15.67	0.47	1200	30.00	0.22	0.82	1500	1.3	3.60	2.50	0.310	0.290	0.600	6.22	5.82
Colombo 01	Moderately weathered Biotite Gneiss	42	66.41	27.36	0.112	3.06	1300	0.96	20.42	0.47	1200	30.00	0.37	0.59	1400	1.2	9.79	2.33	0.280	0.260	0.600	1.02	0.95
Colombo 01	Moderately weathered Biotite Gneiss	42	40.97	16.88	0.112	1.89	1800	2.23	21.50	0.47	900	30.00	0.95	1.28	1000	1.1	15.57	2.22	0.260	0.240	0.600	0.81	0.75
Narammala	Slightly weathered Biotite Gneiss	66	28.24	11.63	0.212	2.47	400	1.20	23.72	0.47	1200	26.00	0.15	0.24	1450	1.2	10.54	2.42	0.270	0.250	0.600	0.17	0.16
Narammala	Moderately weathered Biotite Gneiss	49	150.98	62.20	0.138	8.58	1200	0.30	28.45	0.47	1800	26.00	0.24	0.06	2100	1.2	3.03	2.33	0.330	0.310	0.900	7.64	7.18
Narammala	Moderately weathered Biotite Gneiss	46	140.11	57.73	0.129	7.45	1400	0.54	30.99	0.47	1500	26.00	0.44	0.10	2100	1.4	3.49	2.80	0.310	0.300	0.750	6.01	5.81
Narammala	Slightly weathered Biotite Gneiss	84	150.17	61.87	0.050	40.22	1000	0.15	20.20	0.47	1800	30.00	0.42	0.03	2100	1.2	0.75	2.33	0.590	0.590	0.900	25.52	24.01
Narammala	Slightly weathered Biotite Gneiss	61	61.73	26.75	0.500	18.37	800	0.50	31.90	0.47	1200	26.00	0.51	0.11	2100	1.8	1.42	3.50	0.460	0.440	0.600	6.65	6.36
Narammala	Slightly weathered Biotite Gneiss	80	89.19	36.75	0.188	4.78	2800	0.62	20.65	0.47	1500	30.00	0.40	0.20	2100	1.4	6.27	2.80	0.320	0.280	0.750	11.10	9.71
Narammala	Slightly weathered Biotite Gneiss	40	30.84	20.95	0.112	2.35	1000	0.53	32.10	0.47	1800	30.00	0.20	0.33	2100	1.2	12.79	2.33	0.270	0.240	0.900	0.90	0.80
Narammala	Moderately weathered Biotite Gneiss	53	35.33	22.80	0.150	3.42	900	0.38	20.95	0.47	1800	30.00	0.12	0.26	2100	1.2	8.77	2.33	0.260	0.250	0.900	0.98	0.95
Narammala	Moderately weathered Biotite Gneiss	39	80.19	33.04	0.110	3.63	2800	2.80	21.32	0.47	900	30.00	1.01	0.19	1000	1.1	8.25	2.22	0.310	0.285	0.450	1.62	1.28
Narammala	Moderately weathered Biotite Gneiss	41	46.59	20.97	0.125	3.20	2800	2.80	21.32	0.47	900	30.00	2.12	0.38	1000	1.3	13.84	2.50	0.270	0.240	0.400	0.88	0.68
Narammala	Moderately weathered Biotite Gneiss	44	66.59	26.97	0.125	3.20	2800	2.80	21.32	0.47	900	30.00	2.12	0.38	1000	1.3	13.84	2.50	0.270	0.240	0.400	0.88	0.68
Narammala	Moderately weathered Biotite Gneiss	45	90.33	37.46	0.126	4.23	1800	1.70	20.99	0.47	800	30.00	1.18	0.52	1000	1.3	15.36	2.50	0.330	0.280	0.400	5.84	5.44
Narammala	Moderately weathered Biotite Gneiss	45	91.87	37.85	0.126	4.23	1800	1.70	20.99	0.47	800	30.00	1.18	0.52	1000	1.3	15.36	2.50	0.330	0.280	0.400	5.84	5.44
Narammala	Moderately weathered Biotite Gneiss	41	88.64	36.52	0.096	3.51	3600	4.60	26.58	0.47	800	30.00	3.21	1.39	1000	1.3	8.56	2.50	0.315	0.280	0.400	2.03	1.74
Narammala	Moderately weathered Biotite Gneiss	48	112.82	46.48	0.135	6.26	3200	2.55	20.99	0.47	800	30.00	2.09	0.46	1000	1.3	4.78	2.50	0.310	0.280	0.400	5.42	4.90
Narammala	Moderately weathered Biotite Gneiss	41	49.34	20.33	0.114	2.32	3200	3.40	24.03	0.47	1800	30.00	2.40	0.31	2100	1.2	12.95	2.50	0.240	0.240	0.400	3.76	3.41
Narammala	Moderately weathered Biotite Gneiss	37	54.74	22.55	0.104	2.35	1000	0.40	27.90	0.47	1800	30.00	0.69	0.31	2100	1.2	12.95	2.50	0.260	0.240	0.400	3.76	3.41
Narammala	Slightly weathered Biotite Gneiss	56	67.92	27.98	0.168	4.00	2800	1.41	39.36	0.47	750	30.00	0.98	0.43	1800	1.2	6.38	2.40	0.310	0.275	0.750	2.67	2.37
Narammala	Moderately weathered Biotite Gneiss	50	61.76	25.45	0.089	2.26	500	0.63	14.10	0.47	750	30.00	0.25	0.38	1000	1.3	13.25	2.67	0.260	0.240	0.300	0.76	0.69
Narammala	Moderately weathered Biotite Gneiss	29	39.12	16.12	0.080	1.29	1600	0.68	16.88	0.47	600	30.00	0.30	0.27	1000	1.7	23.27	3.33	0.230	0.210	0.300	0.76	0.69
Narammala	Moderately weathered Biotite Gneiss	53	26.71	11.00	0.150	1.65	1500	2.70	17.40	0.47	1800	30.00	0.07	0.63	2200	1.2	18.17	2.44	0.270	0.240	0.900	0.18	0.16
Narammala	Slightly weathered Biotite Gneiss	83	130.17	53.63	0.050	34.86	1900	0.40	13.33	0.47	1200	30.00	0.35	0.05	2200	1.8	0.86	3.67	0.600	0.580	0.600	38.54	37.25
Narammala	Slightly weathered Biotite Gneiss	85	88.19	36.33	0.162	5.89	1400	0.47	9.75	0.47	1000	26.00	0.31	0.16	2000	2.0	4.42	4.00	0.325	0.300	0.500	5.84	5.12
Narammala	Slightly weathered Biotite Gneiss	53	38.32	15.79	0.150	2.37	500	0.25	7.40	0.47	1500	26.00	0.04	0.21	1800	1.2	10.98	2.40	0.280	0.250	0.750	0.88	0.79
Narammala	Slightly weathered Biotite Gneiss	58	99.33	40.92	0.175	7.16	500	0.37	25.70	0.47	900	30.00	0.32	0.05	1000	1.1	4.19	2.22	0.290	0.280	0.450	6.00	5.80
Narammala	Slightly weathered Biotite Gneiss	61	76.67	31.59	0.188	5.94	200	0.22	15.53	0.47	750	30.00	0.17	0.05	1000	1.7	5.05	3.33	0.310	0.290	0.300	4.31	4.03
Narammala	Moderately weathered Biotite Gneiss	46	67.69	27.77	0.128	3.04	800	0.90	14.80	0.47	750	30.00	0.42	0.48	1000	1.3	9.86	2.67	0.280	0.260	0.400	1.24	1.07
Narammala	Moderately weathered Biotite Gneiss	58	119.31	49.16	0.175	8.60	2200	0.90	30.70	0.47	800	26.00	0.69	0.21	1750	1.2	3.02	2.33	0.330	0.320	0.750	4.62	4.48
Narammala	Moderately weathered Biotite Gneiss	49	31.19	12.85	0.138	1.77	450	1.00	17.80	0.47	800	30.00	0.25	0.75	1000	1.3	16.92	2.50	0.270	0.240	0.400	0.40	0.36
Narammala	Moderately weathered Biotite Gneiss	49	32.39	12.85	0.138	1.77	450	1.00	17.80	0.47	800	30.00	0.25	0.75	1000	1.3	16.92	2.50	0.270	0.240	0.400	0.40	0.36
Narammala	Moderately weathered Biotite Gneiss	49	32.39	12.85	0.138	1.77	450	1.00	17.80	0.47	800	30.00	0.25	0.75	1000	1.3	16.92	2.50	0.270	0.240	0.400	0.40	0.36
Narammala	Moderately weathered Biotite Gneiss	51	33.32	13.72	0.150	1.83	450	0.35	13.00	0.47	1800	26.00	0.06	0.72	2100	1.2	12.41	2.50	0.265	0.245	0.400	0.32	0.30
Narammala	Moderately weathered Biotite Gneiss	51	33.32	13.72	0.150	1.83	450	0.35	13.00	0.47	1800	26.00	0.06	0.72	2100	1.2	12.41	2.50	0.265	0.245	0.400	0.32	0.30
Narammala	Moderately weathered Biotite Gneiss	69	123.98	51.74	0.227	11.59	2400	0.33	9.67	0.47	1500	30.00	0.21	0.12	1900	1.2	2.59	2.53	0.350	0.335	0.750	15.91	15.45
Narammala	Moderately weathered Biotite Gneiss	72	134.42	55.38	0.232	12.85	3600	0.34	9.67	0.47	1800	26.00	0.25	0.09	2100	1.2	2.02	2.33	0.370	0.360	0.900	9.05	8.62
Narammala	Slightly weathered Biotite Gneiss	72	134.48	55.41	0.232	12.85	3600	0.34	9.67	0.47	1800	26.00	0.25	0.09	2100	1.2	2.02	2.33	0.370	0.360	0.900	15.92	15.52
Narammala	Slightly weathered Biotite Gneiss	83	140.28	57.80	0.050	37.57	800	0.08	13.68	0.47	1800	30.00	0.07	0.01	2000	1.1	2.33	2.22	0.600	0.600	0.900	7.44	7.24
Narammala	Moderately weathered Biotite Gneiss	53	119.38	49.18	0.150	7.38	400	0.14	21.85	0.47	1200	30.00	0.12	0.02	2000	1.1	4.07	3.33	0.340	0.280	0.900	42.16	40.76
Narammala	Slightly weathered Biotite Gneiss	80	103.45	42.62	0.500	21.31	1000	0.17	22.17	0.47	1800	26.00	0.16	0.16	2000	1.7	4.77	4.07	0.340	0.280	0.600	11.94	9.84
Narammala	Moderately weathered Biotite Gneiss	32	31.10	12.81	0.090	1.15	1000	0.81	30.10	0.47													

Table A2 - Results on the Estimation of Rock Mass Deformability Modulus through Back Analysis (E_{mb}) with all Five Design criteria

Location	Pells & Turner [1]					Kulhawy & Carter [14]					Rowe & Armitage [15]		
	I_p (Shear)	I_p (Complete)	E_{mbs} (GPa)	E_{mbc} (GPa)	E_{mb} (GPa)	I_p (Shear)	I_p (Complete)	E_{mbs} (GPa)	E_{mbc} (GPa)	E_{mb} (GPa)	I_p (Complete)	E_{mbc} (GPa)	E_{mb}^* (GPa)
Colombo 01	0.318	0.305	3.378	3.240	3.24	0.310	0.285	3.293	3.028	3.03	0.600	6.374	4.46
Colombo 01	0.280	0.250	1.085	0.969	0.97	0.270	0.190	1.046	0.736	0.74	0.515	1.996	1.40
Colombo 08	0.300	0.260	1.316	1.140	1.14	0.280	0.235	1.228	1.031	1.03	0.580	2.544	1.78
Alawwa	0.350	0.310	11.567	10.245	10.25	0.330	0.275	10.906	9.088	9.09	0.720	23.795	16.66
Alawwa	0.310	0.260	0.832	0.698	0.70	0.282	0.225	0.757	0.604	0.60	0.560	1.504	1.05
Colombo 01	0.320	0.300	2.717	2.547	2.55	0.300	0.265	2.547	2.250	2.25	0.610	5.180	3.63
Colombo 01	0.315	0.300	1.524	1.451	1.45	0.302	0.268	1.461	1.297	1.30	0.620	3.000	2.10
Colombo 01	0.325	0.280	1.545	1.331	1.33	0.328	0.272	1.560	1.293	1.29	0.710	3.376	2.36
Colombo 01	0.230	0.200	0.422	0.367	0.37	0.230	0.175	0.422	0.321	0.32	0.410	0.752	0.53
Colombo 01	0.310	0.290	6.223	5.822	5.82	0.380	0.285	7.629	5.721	5.72	0.780	15.659	10.96
Colombo 01	0.280	0.260	1.024	0.951	0.95	0.288	0.205	1.053	0.750	0.75	0.540	1.975	1.38
Colombo 01	0.260	0.240	0.814	0.752	0.75	0.278	0.190	0.871	0.595	0.59	0.480	1.503	1.05
Narammala	0.270	0.250	0.172	0.159	0.16	0.280	0.196	0.178	0.125	0.12	0.500	0.318	0.22
Narammala	0.330	0.310	7.642	7.179	7.18	0.380	0.288	8.800	6.670	6.67	0.800	18.527	12.97
Narammala	0.310	0.300	6.005	5.811	5.81	0.360	0.280	6.974	5.424	5.42	0.790	15.303	10.71
Alawwa	0.590	0.555	25.521	24.007	24.01	0.530	0.480	22.926	20.763	20.76	1.000	43.257	30.28
Alawwa	0.460	0.440	6.648	6.358	6.36	0.470	0.385	6.792	5.564	5.56	0.960	13.873	9.71
Colombo 02	0.320	0.280	11.103	9.715	9.71	0.298	0.255	10.339	8.847	8.85	0.615	21.338	14.94
Alawwa	0.270	0.240	0.902	0.802	0.80	0.274	0.198	0.916	0.662	0.66	0.450	1.504	1.05
Colombo 01	0.260	0.250	0.985	0.947	0.95	0.280	0.215	1.061	0.815	0.81	0.570	2.159	1.51
Colombo 07	0.310	0.245	1.617	1.278	1.28	0.282	0.216	1.471	1.127	1.13	0.585	3.052	2.14
Colombo 07	0.300	0.260	3.075	2.665	2.67	0.284	0.214	2.911	2.194	2.19	0.565	5.792	4.05
Colombo 07	0.270	0.240	5.443	4.838	4.84	0.272	0.197	5.483	3.971	3.97	0.430	8.668	6.07
Colombo 07	0.330	0.280	2.840	2.410	2.41	0.296	0.264	2.547	2.272	2.27	0.610	5.250	3.67
Colombo 07	0.325	0.275	4.295	3.634	3.63	0.299	0.265	3.951	3.502	3.50	0.612	8.087	5.66
Colombo 07	0.315	0.270	2.033	1.743	1.74	0.283	0.226	1.826	1.459	1.46	0.561	3.621	2.53
Colombo 07	0.310	0.280	5.422	4.898	4.90	0.326	0.273	5.702	4.775	4.78	0.715	12.507	8.75
Colombo 07	0.270	0.240	2.151	1.912	1.91	0.272	0.194	2.167	1.545	1.55	0.430	3.425	2.40
Colombo 07	0.265	0.240	3.760	3.406	3.41	0.273	0.197	3.874	2.796	2.80	0.450	6.386	4.47
Colombo 07	0.310	0.275	2.671	2.369	2.37	0.297	0.264	2.559	2.274	2.27	0.610	5.255	3.68
Wellampitiya	0.240	0.225	0.842	0.789	0.79	0.272	0.186	0.954	0.652	0.65	0.460	1.614	1.13
Colombo 08	0.230	0.210	0.761	0.694	0.69	0.268	0.182	0.886	0.602	0.60	0.425	1.406	0.98
Kurunagala	0.270	0.240	0.183	0.162	0.16	0.270	0.185	0.183	0.125	0.13	0.470	0.318	0.22
Kurunagala	0.600	0.580	38.538	37.253	37.25	0.526	0.480	33.785	30.830	30.83	1.000	64.230	44.96
Kurunagala	0.325	0.285	5.835	5.117	5.12	0.350	0.285	6.284	5.117	5.12	0.710	12.747	8.92
Kerawalapitiya	0.280	0.250	0.880	0.786	0.79	0.276	0.191	0.867	0.600	0.60	0.490	1.540	1.08
Katubedda	0.290	0.280	6.003	5.796	5.80	0.336	0.275	6.955	5.692	5.69	0.725	15.007	10.51
Katubedda	0.310	0.290	4.308	4.031	4.03	0.320	0.280	4.447	3.892	3.89	0.610	8.478	5.93
Wellampitiya	0.280	0.240	1.244	1.066	1.07	0.286	0.202	1.270	0.897	0.90	0.485	2.154	1.51
Peliyagoda	0.330	0.320	4.623	4.483	4.48	0.380	0.290	5.324	4.063	4.06	0.800	11.208	7.85
Kerawalapitiya	0.270	0.240	0.405	0.360	0.36	0.268	0.183	0.402	0.274	0.27	0.465	0.697	0.49
Kerawalapitiya	0.265	0.245	0.320	0.296	0.30	0.270	0.187	0.326	0.226	0.23	0.475	0.574	0.40
Ambeupussa	0.600	0.580	25.348	24.503	24.50	0.527	0.482	22.264	20.363	20.36	1.000	42.247	29.57
Ambeupussa	0.270	0.240	0.450	0.400	0.40	0.274	0.198	0.457	0.330	0.33	0.485	0.808	0.57
Colombo 13	0.350	0.335	9.008	8.622	8.62	0.400	0.310	10.295	7.979	7.98	0.860	22.134	15.49
Colombo 13	0.370	0.360	15.948	15.517	15.52	0.425	0.312	18.318	13.448	13.45	0.870	37.499	26.25
Colombo 13	0.370	0.360	7.443	7.242	7.24	0.410	0.298	8.248	5.995	5.99	0.865	17.401	12.18
Mirigama	0.600	0.580	42.164	40.758	40.76	0.515	0.485	36.190	34.082	34.08	1.000	70.273	49.19
Ambeupussa	0.340	0.280	11.943	9.835	9.84	0.386	0.278	13.559	9.765	9.77	0.720	25.291	17.70
Mirigama	0.560	0.470	49.491	41.537	41.54	0.471	0.470	41.625	41.537	41.54	0.970	85.725	60.01
Ambeupussa	0.270	0.240	0.480	0.427	0.43	0.260	0.180	0.462	0.320	0.32	0.475	0.845	0.59
Mirigama	0.270	0.240	0.145	0.129	0.13	0.258	0.175	0.139	0.094	0.09	0.465	0.250	0.17
Ambeupussa	0.280	0.250	0.783	0.699	0.70	0.266	0.181	0.744	0.506	0.51	0.490	1.370	0.96
Colombo 07	0.300	0.260	1.836	1.591	1.59	0.286	0.202	1.750	1.236	1.24	0.500	3.060	2.14
Colombo 02	0.310	0.270	1.762	1.535	1.54	0.310	0.245	1.762	1.393	1.39	0.615	3.496	2.45
Colombo 02	0.320	0.310	4.027	3.901	3.90	0.320	0.278	4.027	3.498	3.50	0.715	8.997	6.30
Colombo 02	0.330	0.315	4.026	3.843	3.84	0.321	0.279	3.916	3.403	3.40	0.710	8.661	6.06
Colombo 02	0.320	0.310	6.066	5.877	5.88	0.374	0.288	7.090	5.460	5.46	0.765	14.502	10.15
Colombo 01	0.260	0.210	0.472	0.381	0.38	0.230	0.173	0.417	0.314	0.31	0.380	0.690	0.48
Colombo 01	0.300	0.280	18.174	16.962	16.96	0.323	0.278	19.567	16.841	16.84	0.713	43.194	30.24
Colombo 01	0.260	0.200	0.890	0.685	0.68	0.245	0.186	0.839	0.637	0.64	0.425	1.455	1.02
Colombo 01	0.270	0.210	0.741	0.577	0.58	0.250	0.190	0.687	0.522	0.52	0.430	1.181	0.83
Colombo 01	0.310	0.275	2.803	2.487	2.49	0.305	0.265	2.758	2.396	2.40	0.620	5.607	3.92
Colombo 01	0.315	0.270	4.626	3.965	3.96	0.323	0.270	4.743	3.965	3.96	0.713	10.471	7.33
Colombo 01	0.290	0.265	1.827	1.670	1.67	0.280	0.265	1.764	1.670	1.67	0.540	3.402	2.38
Colombo 01	0.260	0.220	0.745	0.631	0.63	0.272	0.186	0.780	0.533	0.53	0.450	1.290	0.90
Colombo 01	0.260	0.230	0.718	0.635	0.63	0.264	0.177	0.729	0.489	0.49	0.440	1.215	0.85
Colombo 02	0.270	0.230	1.074	0.915	0.91	0.268	0.192	1.066	0.764	0.76	0.470	1.869	1.31
Colombo 02	0.320	0.300	5.911	5.541	5.54	0.382	0.279	7.056	5.154	5.15	0.795	14.685	10.28
Colombo 02	0.310	0.290	2.759	2.581	2.58	0.299	0.266	2.661	2.367	2.37	0.630	5.606	3.92
Colombo 02	0.310	0.290	2.572	2.406	2.41	0.298	0.268	2.473	2.224	2.22	0.624	5.178	3.62
Colombo 02	0.430	0.400	43.757	40.705	40.70	0.450	0.368	45.793	37.448	37.45	0.910	92.603	64.82
Colombo 02	0.340	0.320	2.442	2.299	2.30	0.375	0.281	2.694	2.019	2.02	0.800	5.747	4.02
Colombo 02	0.260	0.220	0.672	0.569	0.57	0.266	0.194	0.688	0.502	0.50	0.460	1.189	0.83

